2022

(May/June)

ECONOMICS

(Honours)

(Mathematics for Economists)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Distinguish between a function and a relation. Explain some of the different functions and their uses in Economics. 4+6=10

(b) State and prove the associative law of set operations using the following sets : 5

$A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 5, 8, 9\}$
$C = \{6, 8, 9\}$

2. (a) The straight line passes through the point $(1, -2)$ and makes the intercept on the x-axis which is double the intercept on the y-axis. Find the equation of the line. Also write down the gradient of the line. 4+1=5

(b) Find the equilibrium price and quantity for the following market model : 5

$$Q_d = 20 - 3P$$
$$Q_s = -5 + 2P$$

(c) Explain the difference between homogeneous and homothetic functions with examples. 5

UNIT—II

3. (a) Define the meaning of a 'skew symmetric' matrix with a suitable example. 2

(b) For any 2×2 matrices A and B, prove that—

(i) $(A+B)' = A' + B'$

(ii) $(AB)' = B'A'$

(iii) $|A||B| = |AB|$ 3+3+3=9

(c) If

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

find $A^2 - 5A + 7I$. 4

4. (a) Solve the following simultaneous equations using the matrix inversion method : 9

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

(b) In the Leontief input-output model, explain the concept of the input-output table and the use of the technical coefficient matrix. 3+3=6

UNIT—III

5. *(a)* Define 'limit of a variable' and 'limit of a function'. 3

(b) Evaluate the limit of the following functions : 2+2+2=6

(i) $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

(ii) $\lim_{x \to 0} \frac{\sqrt{2-x}-\sqrt{2+x}}{x}$

(iii) $\lim_{x \to -} \frac{(x-1)(2x+3)}{(x+2)(3x+4)}$

(c) Given

$$f(x) = 4x+3 \quad \text{for} \quad x > 4 \text{ and } x < 4$$
$$= 3x+7 \quad \text{for} \quad x = 4$$

Examine its continuity at $x = 4$. 6

6. *(a)* Find $\frac{dy}{dx}$ of the following (any *four*) : 2×4=8

(i) $y = \frac{2x^3 - x^2 + x - 2}{x^2}$

(ii) $y = e^{\sqrt{x^2+3x-4}}$

(iii) $y = (x^2+3)(2x^2+7)^3$

(iv) $4x^2 + 2xy + y^2 = 12$

(v) $y = x^{x+1}$

(b) Given $z = x^3 e^{2y}$, find all the partial derivatives of the second-order and prove that $f_{xy} = f_{yx}$. 4

(c) If $u(x, y) = \log(x+y)$, then find the total differential of u. 3

UNIT—IV

7. *(a)* Briefly explain the concept of maxima and minima for a given function $y = f(x)$. Also explain the point of inflection with the help of a suitable example. 4+3=7

(b) Find the maximum and minimum values of the function $y = 4x + \frac{1}{x}$. 4

(c) If total cost $C = 4x^3 - 3x^2 + 200x$, find the slope of both the AC curve and the MC curve when $x = 2$. Also interpret the meaning of the slope. 3+1=4

8. *(a)* In a perfectly competitive market, the total cost of a firm is given by $TC = Q^2 - 6Q + 10$ and the price of the product is 4 per unit. Find the profit maximising output. What are the corresponding values of MC and MR? 5+2+2=9

(b) For the average revenue function, $AR = 20 - 2Q$, find the output level at which TR is maximum. Also show that elasticity of demand is equal to unity at this output level. 4+2=6

UNIT—V

9. *(a)* What is integration? Explain its uses in Economics. 1+2=3

(b) Find the integral of the following (any *four*) : 3×4=12

(i) $\int \frac{2x+2}{(x^2+2x-10)^3}\,dx$

(ii) $\int \sqrt{x} \log x\,dx$

(iii) $\int \sqrt[3]{3x+8}\,dx$

(iv) $\int \frac{x^2-2}{(x-1)(x-2)}\,dx$

(v) $\int (x+2)e^{5x}dx$

(Continued)

10. *(a)* Prove that

$$\int_1^3 (4x - x^2 - 3)\,dx = \frac{4}{3}$$ 3

(b) Given demand function $Q = \sqrt{60 - \frac{3}{2}P}$.

Obtain consumer surplus when $P = 16$. 6

(c) If $D = 250 - 50P$ and $S = 25P + 25$ are the demand and the supply functions respectively, find the producer's surplus under equilibrium. 6

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